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LINEAR DIFFERENTIAL EQUATIONS.

Handbuch der Theorie der linearen Differentialgleichungen. Von Prof. Dr. Ludwig Schlesinger, Privatdocenten an der Universität zu Berlin. Erster Band. (Leipzig: Teubner, 1895.)

DE MORGAN is reported to have said of the subject of differential equations, that it illustrated the proverb that he who hides knows how to find. This was true enough at a time when the sole aim of the analyst was to "solve" differential equations by reducing them to quadratures, or to construct ingenious puzzles for the benefit of undergraduates. Integration by series was known, of course; but this was regarded as a mean device, useful indeed for purposes of calculation, especially to the physicist, but unworthy of the serious attention of the pure mathematician.

A new era began with the foundation of what is now called function-theory by Cauchy, Riemann, and Weierstrass. The study and classification of functions according to their essential properties, as distinguished from the accidents of their analytical forms, soon led to a complete revolution in the theory of differential equations. It became evident that the real question raised by a differential equation is not whether a solution, assumed to exist, can be expressed by means of known functions, or integrals of known functions, but in the first place whether a given differential equation does really suffice for the definition of a function of the independent variable (or variables), and, if so, what are the characteristic properties of the function thus defined. Few things in the history of mathematics are more remarkable than the developments to which this change of view has given rise. Leaving out of account the theory of partial differential equations, which is still beset with many and serious difficulties, it is not too much to say that in the course of less than half a century the theory of ordinary linear differential equations alone has attained a degree of extent and importance which makes it comparable with almost any of the main branches of analysis.

The landmarks of the new departure are the memoir of Briot and Bouquet in the *Journal de l'École Polytechnique* (cap. 36), Riemann's paper on the generalised hypergeometric series, and Fuchs's memoir in the sixty-sixth volume of *Crelle's Journal*. Since the publication of this last work, more especially, the progress made has been exceedingly rapid: the general principles of the subjects have been permanently established, so as already to admit of methodical treatment, and numerous particular applications, all of great interest and beauty, have attracted and continue to invite the attention of mathematical explorers. Thus there is the problem of discovering whether a given equation has an algebraic integral, and, if so, of finding it; there is the theory of equations with doubly periodic coefficients; and there is the theory of differential invariants. Each of these is associated with some of the most brilliant discoveries of modern analysis, and each offers abundant opportunity for further research.

The wide extent of the subject, and the immense number of memoirs relating to it, have created an urgent need for systematic treatises to serve as an introduction to the theory, and presenting its main outlines in a proper perspective. Fortunately this want seems likely to be supplied before long; various excellent works, dealing wholly or in part with linear differential equations, have recently appeared or are in course of publication, and among these the book now under review will take an honourable place.

Dr. Schlesinger's work, to be completed in two volumes, is intended to give a coherent and comprehensive account of the theory in the light of its most recent developments. This first volume is divided into eight sections, exclusive of two introductory chapters, one historical, the other treating of the existence of an integral, and the general nature of the singular points. Of the eight sections, the first contains the first principles of the theory, mostly after Fuchs; the second discusses systems of independent integrals, reduction when particular integrals are known, Lagrange's adjoint equation, non-homogeneous equations, and Frobenius's theorems on irreducibility; the third relates to the fundamental equation; the fourth to unessential singular points; the fifth to equations of the "Fuchsian" class, that is to say, of which the coefficients are rational functions of x and all the integrals are regular; the sixth treats of the development of integrals within a circular annulus; and, finally, the seventh and eighth contain the general theory of equations with rational coefficients.

The treatment is entirely analytical, and is based principally on the methods of Weierstrass as expounded by Fuchs, Frobenius, Hamburger and others; thus the integrals are obtained in the form of power-series valid within a certain region of the plane of the complex variable, and no use is made of geometrical diagrams such as those employed by Schwarz, Klein, and Goursat. Moreover, except in the fifth section, which contains a brief discussion of Riemann's P-function and of the hypergeometric series, the author confines himself to the *general* theory, and does not consider special cases, or particular applications. The demonstrations, for the most part, are concise, and free use is made of the sign of summation and double suffixes. For these reasons the book is perhaps hardly suitable for those who are approaching the subject for the first time; but any one who has read, let us say, Goursat's thesis on the hypergeometric series, or Klein's lectures on linear differential equations of the second order, and is moderately familiar with the Weierstrassian function-theory, will be able to consult it with advantage. To those who are engaged in research, Dr. Schlesinger's treatise will be of great value, because those parts of the subject which are included within the author's plan are discussed with sufficient thoroughness, with a consistent notation, and in logical order; while the analytical table of contents gives references to the original sources in direct connection with the articles of the book which are based upon them. It is rather a pity, by-the-by, that the dates have not always been given in these references; the reader may very possibly wish to know the date of a paper, and not be able to consult the volume of the journal in which it appeared.

Mathematicians will look forward with interest to

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the appearance of the concluding volume of the treatise, which will contain, *inter alia*, a discussion of the group of an equation, and of the classification of equations according to the nature of the groups belonging to them. Until the work is complete, it is premature to express an opinion as to the degree of success with which the author has attained the object he has in view; but there can be no doubt of the valuable service which is rendered to science by the composition of a methodical treatise like this. So far as we are able to judge, account has been taken of all the most important researches which come within the scope of the present volume; the three last sections, in particular, include an account of the recently published papers of Helge von Koch, Poincaré, and Mittag-Leffler.

The proof-sheets appear to have been very carefully revised, so that the book is happily free from the crowd of misprints with which mathematical text-books, otherwise excellent, are not unfrequently disfigured. G. B. M.

THE RESEARCHES OF TESLA.

Inventions, Researches, and Writings of Nikola Tesla. By Thomas Commerford Martin. (New York: *The Electrical Engineer*, 1894.)

WE have here an account of Nikola Tesla, his scientific inventions and work, by a devoted admirer. Mr. Martin is not a Boswell, and from the nature of the case his book could hardly have about it all that human interest which pervades the life and achievements of a veteran discoverer in science. Mr. Tesla is a young man whose career has been somewhat romantic, and whose ingenuity is such as to rank him very high indeed among the electrical workers and discoverers of the day. Born in Austro-Hungary, educated at the Realschule at Carstatt and the Polytechnic at Gratz, and professionally first in the Government Telegraph Department, and afterwards in Paris, his career as an engineer really began when he arrived in America little more than ten years ago.

In two or three years from the day on which he took off his coat in the Edison Works, Tesla motors had attracted attention, and he leaped at once to a position as a successful experimenter and inventor, which his subsequent work has only secured and made more important. His researches on the effects of alternating currents of high potential and frequency, in particular, though they had the misfortune to be made the subject of the speculations of the ordinary journalist, are of great scientific interest, and continued by Mr. Tesla himself and the army of enthusiastic workers we now have, cannot fail to yield theoretical results and practical applications which will more than fulfil the anticipations of those who took a sober and rational view of their possibilities. None of those who listened to Mr. Tesla at the Royal Institution will soon forget the almost marvellous experiments performed, their clear exposition in what was to the lecturer manifestly a foreign language, and the enthusiasm which the results displayed excited in those present who were best able to judge of their scientific interest and importance.

Mr. Martin's account of Mr. Tesla's work is interesting, and yet perhaps it might have been in some respects better than it is. He has had excellent materials, such as the various lectures delivered by Mr. Tesla on his researches generally, the papers read from time to time to scientific societies on particular inventions and points of interest, and apparently the specifications of Mr. Tesla's patents. Our complaint, if we have one, is that this material has hardly been sufficiently worked up. Many of the lectures and papers were, as was inevitable, hurriedly composed, and the expression of Mr. Tesla's theoretical views contained in them is not always so clear and complete as it might have been made by one not so rapidly carried forward by the stream of discovery. A great inventor can hardly be expected to spend time weighing words and phrases, at any rate he has a title to be excused from doing so, which others who expound him do not possess. As it is, Mr. Martin's book is on the whole a reproduction of articles which appeared from time to time in the *Electrical Engineer* (of New York), and all we wish is that he could have spared the time and trouble necessary to cast the matter into a more homogeneous and symmetrical form.

For the lectures which are reproduced we are very grateful. They give Mr. Tesla's own description of his inventions, and his views on points of theory—views, which if not always orthodox, and sometimes expressed in language which appears strange, are always fresh and suggestive. The unavoidable repetitions of the same ideas, and recurring descriptions of the same apparatus, are not without some advantage, though they interfere with the unity of Mr. Martin's book, as they enable the lecturer's meaning to be made out more completely than would otherwise be possible.

The book is divided into four parts: Polyphase currents; Tesla effects with high frequency and high potential currents; miscellaneous inventions and writings; early phase motors and the Tesla oscillators. The two first parts are of course much more interesting than the remaining two, which have to do with such things as oil condensers, anti-sparking dynamo brushes, unipolar generators, the Tesla exhibit at the World's Fair, and the Tesla mechanical and electrical oscillators.

The discussion of polyphase currents, which occupies the first 115 pages of the book, has more unity of treatment about it than the second part, which consists mainly of the lectures Mr. Tesla delivered in this country and America. After a short introductory and biographical chapter, Mr. Martin proceeds to expound the principle of the rotating magnetic field and the construction of synchronising motors. A paper by Tesla, on a "New System of Alternate Current Motors and Transformers," is reproduced in this connection, and contains the foundation on which is based the remaining twenty-one chapters which make up Part i. These contain numerous modifications of the original idea, many of them exceedingly ingenious. A motor "depending on 'magnetic lag' or hysteresis" is described in Chapter xii. The peculiarity of this is stated in an introductory paragraph to be "that in it the attractive effects or phases, while lagging behind the phases of current which produce them, are manifested simultaneously and not successively." This statement itself seems to want some little exposition, though the arrangement is really very simple. An iron disc is pivoted within a fixed coil, wound just

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